Trading networks with bilateral contracts*

[Extended Abstract]†

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ABSTRACT

The concept of stability in matching markets is an important predictor of outcomes. We study production networks in which firms match and sign bilateral contracts. A firm acts either as a buyer or a seller in any particular contract. The case when production networks are supply chains (i.e. when a firm cannot buy from and sell to another firm even via intermediaries) has been studied extensively [4, 5, 2]. In supply chains, chain-stable outcomes are guaranteed to exist and they coincide with group-stable outcomes. We study production networks in which firms can buy from and sell to one another directly or via intermediaries i.e. contracts may form a cycle. It is well known that in this case group-stable outcomes might not exist [1, 2]. We show that the problem of determining whether an allocation is group-stable is NP-hard. We define a new stability concept, called *trail stability*, and show that any network of bilateral contracts has a trail-stable outcome whenever agents' preferences satisfy full substitutability [4, 2]. Trail-stable outcomes rule out consecutive and consistent pairwise blocks that form trails of contracts (sequences of distinct contracts in which each intermediary who is a buyer in one contract is a seller in the next one). Trail stability is a natural extension of chain stability and is a stronger solution concept in general contract networks. Trail-stable outcomes may not be immune to group deviations or efficient. In fact, we show that outcomes satisfying an even more demanding stability property – *full trail stability* – always exist. Fully trail-stable outcomes also rule out trail blocks, but an intermediary is not required to choose all contracts in the trail – only local upstream-downstream pairs. We pin down conditions under which terminal contracts (i.e. involving agents who sign either only downstream or only upstream contracts) in trail-stable and fully trail-stable outcomes have a lattice structure. We describe the relationships between all stability concepts. When contracts specify trades and prices, we also show that trail-stable competitive equilibrium outcomes exist in networked markets even when agents' utility functions are not quasilinear, extending the results of [3].

Categories and Subject Descriptors

[**Applied Computing**]: Law, social and behavioral sciences – Economics

General Terms

Algorithms, Economics, Theory

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